Microscopic Calculation of the Total Cross Section for the $^6Li(n, \alpha)^3H$ Transfer Reaction

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Motivation

Cross sections of nuclear reactions induced by light nuclei at low energies are necessary for solving many physical problems, especially in nuclear astrophysics.

As a rule, these cross sections either have large experimental errors or turn out to be inaccessible for reliable measurements in the energy range being of interest to the investigations.

Purely mathematical extrapolations of the experimental data from high energy range to necessary low energies are often unreliable.

That is why theoretical calculations based on microscopic approaches are a very important source of knowledge of the cross sections.

As a result, the building such approaches is a significant challenge.

The main aim of the present work is to demonstrate the results of study of the $^{6}Li(n,\alpha)^{3}H$ transfer reaction within a developed microscopic approach based on the algebraic version of the resonating group model.

Microscopic approach

Physical viewpoint:

- Description of a dynamics of all nucleons of a nuclear system;
- Complete account of the Pauli exclusion principle;
- Rigorous treatment of the center-of-mass motion.

From the mathematical viewpoint, the wave functions have the features:

- Explicit dependence on space and spin-isospin coordinates of all the nucleons of the system;
- Full antisymmetrization over all pair nucleon permutations;
- Translational invariance.

The algebraic version of the resonating group model (AVRGM)

The total wave function of a system within the multichannel RGM is written as

$$\Psi = \sum_i \mathrm{A}\left\{\varphi^{(1)_i}\varphi^{(2)_i} f_i(\mathbf{q}_i)\right\}, \qquad \mathbf{q}_i = \sqrt{\frac{A_{1i}A_{2i}}{A_{1i}+A_{2i}}}\left(\mathbf{R}^{(1)_i}_{\text{c.m.}} - \mathbf{R}^{(2)_i}_{\text{c.m.}}\right).$$

The main idea of the AVRGM is to expand the relative motion wave function in series of the oscillator functions:

$$\boxed{f_{\nu lm}(\mathbf{q}) = (-1)^{(\nu-l)/2} N_{\nu l}\,(q/r_0)^l L^{(l+1/2)}_{(\nu-l)/2}(q^2/r_0^2)\exp(-q^2/2r_0^2) Y_{lm}(\mathbf{n}_\mathbf{q})}\ .$$

The total wave function takes the form $\Psi = \sum\limits_{J=J_0}^{\infty}\sum\limits_{M=-J}^{J}\sum\limits_{i}\sum\limits_{s_i=|s_{1i}-s_{2i}|}^{s_{1i}+s_{2i}}\sum\limits_{l_i=|J-s_i|}^{J+s_i}\sum\limits_{\nu_i=\nu_{0i}}^{\infty} C^{(i)}_{J^\pi M l_i s_i \nu_i}\,\Psi^{(i)}_{J^\pi M l_i s_i \nu_i}$

of an expansion over the AVRGM basis functions:

$$\boxed{\Psi^{(i)}_{J^\pi M l_i s_i \nu_i} = N_{J^\pi l_i s_i \nu_i}\,\mathrm{A}\left\{\sum_{m_i+\sigma_i=M} C^{JM}_{l_i m_i\ s_i\sigma_i}\left[\varphi^{(1)_i}_{s_{1i}}\varphi^{(2)_i}_{s_{2i}}\right]_{s_i\sigma_i} f_{\nu_i l_i m_i}(\mathbf{q}_i)\right\}}\ .$$

The expansion coefficients satisfy an infinite set of homogeneous linear algebraic equations:

$$\begin{cases}\sum\limits_{i}\sum\limits_{s_i=|s_{1i}-s_{2i}|}^{s_{1i}+s_{2i}}\sum\limits_{l_i=|J-s_i|}^{J+s_i}\sum\limits_{\nu_i=\nu_{0i}}^{\infty}\left(\left\langle J^\pi M l_j s_j \nu_j j\right|H\left|J^\pi M l_i s_i \nu_i i\right\rangle - E\left\langle J^\pi M l_j s_j \nu_j j\,\middle|\,J^\pi M l_i s_i \nu_i i\right\rangle\right) C^{(i)}_{J^\pi M l_i s_i \nu_i} = 0,\\ i, j = 1, 2, \ldots, \quad s_j = |s_{1j}-s_{2j}|, \ldots, s_{1j}+s_{2j}, \quad l_j = |J-s_j|, \ldots, J+s_j, \quad \nu_j = \nu_{0j}, \nu_{0j}+2, \ldots\ .\end{cases}$$

The AVRGM Equations Set for Continuum

Taking into account the boundary conditions for continuum, the AVRGM equations set can be reduced to a finite set of inhomogeneous linear algebraic equations:

$$\begin{cases} \sum\limits_{i} \sum\limits_{s_i=|s_{1i}-s_{2i}|}^{s_{1i}+s_{2i}} \sum\limits_{l_i=|J-s_i|}^{J+s_i} \sum\limits_{\nu_i=\nu_{0i}}^{\nu_{\mathrm{as},i}-2} \left(\langle J^\pi M l_j s_j \nu_j j | H | J^\pi M l_i s_i \nu_i i \rangle - E \langle J^\pi M l_j s_j \nu_j j | J^\pi M l_i s_i \nu_i i \rangle \right) C^{(i)}_{J^\pi M l_i s_i \nu_i} = F_{J^\pi M l_j s_j \nu_j j}, \\ i, j = 1, 2, \ldots, \quad s_j = |s_{1j} - s_{2j}|, \ldots, s_{1j} + s_{2j}, \quad l_j = |J - s_j|, \ldots, J + s_j, \quad \nu_j = \nu_{0j}, \nu_{0j} + 2, \ldots, \nu_{\mathrm{as},j}, \end{cases}$$

$$F_{J^\pi M l_j s_j \nu_j j} = -\sum_{i} \sum_{s_i=|s_{1i}-s_{2i}|}^{s_{1i}+s_{2i}} \sum_{l_i=|J-s_i|}^{J+s_i} \sum_{\nu=\nu_{\mathrm{as},i}}^{\nu'_{\max,i}} \langle J^\pi M l_j s_j \nu_j j | H | J^\pi M l_i s_i \nu_i i \rangle C^{(\mathrm{as})}_{J^\pi M l_i s_i \nu_i i}.$$

Asymptotic expression for the expansion coefficients has the form:

$$C^{(\mathrm{as})}_{J^\pi M l_i s_i \nu_i i} = \delta_{i i_0} \delta_{l_i l_{i_0}} \delta_{s_i s_{i_0}} C^{-}_{l_i \nu_i}(\eta_i, k_i) - S^{J^\pi}_{i l_i s_i,\, i_0 l_{i_0} s_{i_0}} C^{+}_{l_i \nu_i}(\eta_i, k_i),$$

$$C^{\pm}_{l_i \nu_i}(\eta_i, k_i) = \frac{e^{\mp i \sigma_{l_i}}}{\sqrt{k_i q_{0i}}} \left[G_{l_i}(\eta_i, k_i q_{0i}) \pm \mathrm{i} F_{l_i}(\eta_i, k_i q_{0i}) - \frac{k_i^3 r_0^4}{6 q_{0i}} \left(G'_{l_i}(\eta_i, k_i q_{0i}) \pm \mathrm{i} F'_{l_i}(\eta_i, k_i q_{0i}) \right) \right],$$

$$q_0 = r_0 \sqrt{2\nu + 3}, \quad k_i = \sqrt{2 m E^{(i)}_{\text{ц.м.}}} / \hbar, \quad \sigma_{l_i} = \arg \Gamma(l_i + 1 + \mathrm{i}\eta_i), \quad \eta_i = e^2 Z_{1i} Z_{2i} / \hbar \upsilon_i.$$

The Generating Functions Method

The main idea is to utilize the relation between the harmonic oscillator functions and their generating function:

$$\boxed{f_{\nu lm}(\mathbf{q}) = A_{\nu l}\frac{\partial^{\nu}}{\partial R^{\nu}}\int \exp(-q^2/2r_0^2 + \mathbf{qR}/r_0 - R^2/4)Y_{lm}(\mathbf{n_R})d\mathbf{n_R}\Bigg|_{R=0}}$$, $\mathbf{R}$ – generating parameter.

The generating function for the AVRGM basis:

$$\Phi_{s\sigma}^{(1+2)}(\mathbf{R}) = \mathrm{A}\left\{\left[\varphi_{s_1}^{(1)}\varphi_{s_2}^{(2)}\right]_{s\sigma}\exp(-q^2/2r_0^2 + \mathbf{qR}/r_0 - R^2/4)\right\}$$

Relation between the matrix elements in AVRGM basis and the generating matrix elements:

$$\boxed{\left\langle J_f^{\pi_f} M_f l_f s_f \nu_f \left| V \right| J_i^{\pi_i} M_i l_i s_i \nu_i \right\rangle = \frac{1}{\kappa_{\nu_f l_f s_f}\kappa_{\nu_i l_i s_i}\nu_f!\nu_i!}\frac{\partial^{\nu_f}}{\partial Q^{\nu_f}}\frac{\partial^{\nu_i}}{\partial R^{\nu_i}}I_{i\to f}(Q,R)\Bigg|_{R=Q=0}}$$

$$I_{i\to f}(Q,R) = \sum_{m_f\sigma_f m_i\sigma_i} C_{l_f m_f\, s_f\sigma_f}^{J_f M_f} C_{l_i m_i\, s_i\sigma_i}^{J_i M_i}\iint Y_{l_f m_f}^{*}(\mathbf{n_Q})\left\langle \mathbf{Q}, s_f\sigma_f \left| V \right| \mathbf{R}, s_i\sigma_i\right\rangle Y_{l_i m_i}(\mathbf{n_R})d\mathbf{n_Q}d\mathbf{n_R}$$

$$(\nu!)^2\kappa_{\nu ls}^2 = \frac{\partial^{\nu}}{\partial Q^{\nu}}\frac{\partial^{\nu}}{\partial R^{\nu}}\iint Y_{lm}^{*}(\mathbf{n_Q})\left\langle \mathbf{Q}, s\sigma \middle| \mathbf{R}, s\sigma\right\rangle Y_{lm}(\mathbf{n_R})d\mathbf{n_Q}d\mathbf{n_R}\Bigg|_{R=Q=0}$$

Hamiltonian and Nuclear Potential

The Hamiltonian of the system: $H = T - T_{\mathrm{c.m.}} + V_{\mathrm{coul}} + V_{\mathrm{nucl}}^{(\mathrm{H\text{-}N})}$

$$T - T_{\mathrm{c.m.}} = -\frac{\hbar^2}{2mA}\sum_{i>j=1}^{A}\left(\nabla_i - \nabla_j\right)^2$$ – kinetic energy

$$V_{\mathrm{coul}} = \sum_{i>j=1}^{Z}\frac{e^2}{r_{ij}}$$ – Coulomb interaction of the protons

The modified Hasegawa–Nagata potential (MHN potential):

$$V_{\mathrm{nucl}}^{(\mathrm{H\text{-}N})} = \sum_{i>j=1}^{A}\left(V_{ij}^{(\mathrm{c})} + V_{ij}^{(ls)} + V_{ij}^{(\mathrm{t})}\right)$$

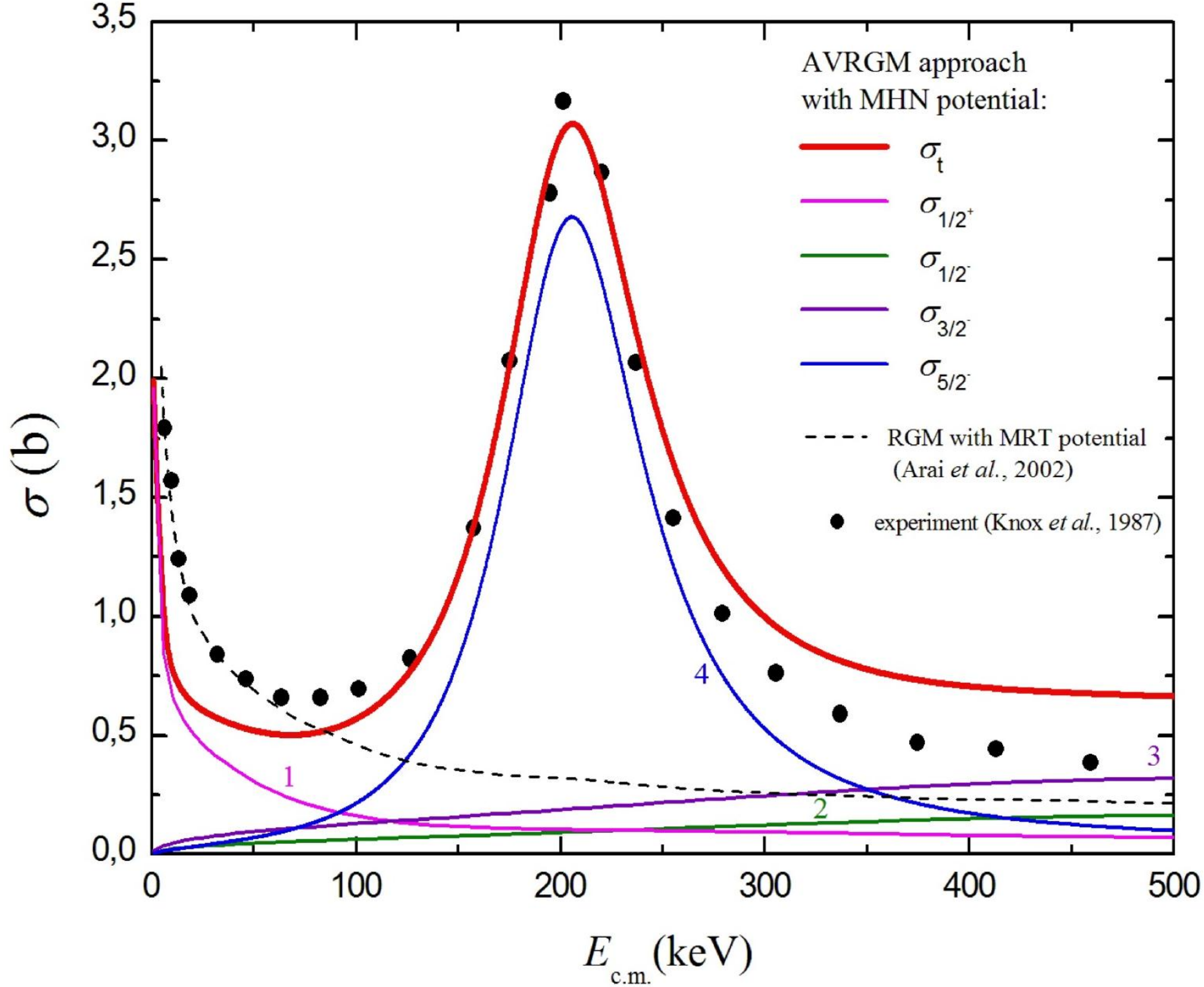
$$V_{ij}^{(\mathrm{c})} = \sum_{n=1}^{3} V_n^{(\mathrm{c})}\left(w_n^{(\mathrm{c})} + (1-g_c)m_n^{(\mathrm{c})} - g_c m_n^{(\mathrm{c})} P_{ij}^{\sigma} P_{ij}^{\tau} + b_n^{(\mathrm{c})} P_{ij}^{\sigma} - h_n^{(\mathrm{c})} P_{ij}^{\tau}\right)\exp\left(-\mu_n^{(\mathrm{c})} r_{ij}^2\right)$$

$$V_{ij}^{(ls)} = g_{ls}\sum_{n=1}^{2}\frac{V_n^{(ls)}}{2}\left(w_n^{(ls)} - h_n^{(ls)} P_{ij}^{\tau}\right)\left[\mathbf{r}_{ij}\times(\mathbf{p}_i - \mathbf{p}_j)\right](\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\exp\left(-\mu_n^{(ls)} r_{ij}^2\right)$$

$$V_{ij}^{(\mathrm{t})} = g_{\mathrm{t}}\sum_{n=1}^{3} V_n^{(\mathrm{t})}\left(w_n^{(\mathrm{t})} - h_n^{(\mathrm{t})} P_{ij}^{\tau}\right)\left(3(\boldsymbol{\sigma}_i\mathbf{r}_{ij})(\boldsymbol{\sigma}_j\mathbf{r}_{ij}) - (\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j)\mathbf{r}_{ij}^2\right)\exp\left(-\mu_n^{(\mathrm{t})} r_{ij}^2\right)$$

Total and partial cross sections for the ^{6}Li(n, α)^{3}H reaction

Cross section for a transfer reaction: $$\sigma_{i\to f}(E_{\text{c.m.}}) = \frac{\pi}{k^2}\sum_{J^\pi}\frac{2J+1}{(2s_1+1)(2s_2+1)}\sum_{l_i l_f s_i s_f}\left|S^{J^\pi}_{il_is_i,\,fl_fs_f}(E_{\text{c.m.}})\right|^2$$



σ_t : $J^\pi = 1/2^\pm, \ldots, 9/2^\pm$.

1 : $J^\pi = 1/2^+$,
^{6}Li + n, $s = 1/2$, $l = 0$ →
α + t, $s = 1/2$, $l = 0$.
(“1/υ” law)

2 : $J^\pi = 1/2^-$,
^{6}Li + n, $s = 1/2$, $l = 1$ →
α + t, $s = 1/2$, $l = 1$.

3 : $J^\pi = 3/2^-$,
^{6}Li + n, $s = 1/2$, $l = 1$ →
α + t, $s = 1/2$, $l = 1$.

4 : $J^\pi = 5/2^-$,
^{6}Li + n, $s = 3/2$, $l = 1$ →
α + t, $s = 1/2$, $l = 3$.
(resonant behavior in vicinity of ~ 200 keV)

Conclusions

1. The $^{6}\mathrm{Li}(n, \alpha)^{3}\mathrm{H}$ reaction has been studied in the framework of the microscopic approach based on the multichannel AVRGM, using the MHN potential to describe nuclear interaction.
2. The total and partial cross sections for the reaction have been calculated at low energies (< 500 keV).
3. The contributions of the dominating transitions between the reaction channels to the energy dependence of the total cross section have been shown.
4. The comparison of the obtained results with the experimental data and the RGM calculation by other authors using the MRT potential has been performed. It demonstrates a reasonable agreement with the data and advantages over the RGM calculation.

Thank you for attention